Name: ____________________

SM3 13.3 NH: Series Notation & Geometric Series

A series is the value you get when you sum the terms of a sequence. We use the Greek letter sigma (Σ) to write sigma notation to indicate which terms of a sequence we're going to add up.

$$\sum_{k=1}^{n}$$ means that the series begins with the variable equal to 1 and ends when the variable is n.

$$\sum_{k=1}^{n} f(k)$$ is calculated as $f(1) + f(2) + f(3) + \cdots + f(n)$

Example: Expand the series and find the sum of $\sum_{k=1}^{4} 6k - 1$

$$\sum_{k=1}^{4} 6k - 1$$

$$= (6 \cdot 1 - 1) + (6 \cdot 2 - 1) + (6 \cdot 3 - 1) + (6 \cdot 4 - 1)$$

$$= 5 + 11 + 17 + 23$$

$$= 56$$

When we don't have an explicit rule for the sequence, we have to build that before we can figure out the series.

Example: Write the series using sigma notation: $c = \frac{5}{8}, \frac{5}{4}, \dots, 80$

The sequence appears to be geometric (multiplicative) because the second value is twice the first.

We'll need to use the geometric sequence formula: $a_n = a_1 \cdot r^{n-1}$ with $r = 2$, $a_1 = \frac{5}{8}$, and $n = 8$.

$$\sum_{k=1}^{8} \frac{5}{8} \cdot 2^{k-1}$$ Since the coefficient of the exponential has a portion that is also a power of our base, we can combine them and further simplify the series expression as follows.

$$\sum_{k=1}^{8} 5 \cdot 2^{-3} \cdot 2^{k-1}$$ An 8 in the denominator is the same as 2^{-3}.

$$\sum_{k=1}^{8} 5 \cdot 2^{k-4}$$ Add exponents when multiplying two expressions with the same base.

We'd like to be able to evaluate geometric series in a faster manner than expanding the series and adding up each term.

$s =$	$\displaystyle\sum_{k=1}^{n} a_1 \cdot r^{k-1}$	Define s to be the sum of the series.
$s =$	$a_1 + a_1r + a_1r^2 + a_1r^3 + \cdots + a_1r^{n-1}$	Expand and sum the geometric series
$s - rs =$	$(a_1 + a_1r + a_1r^2 + a_1r^3 + \cdots + a_1r^{n-1}) +$ $-r(a_1 + a_1r + a_1r^2 + a_1r^3 + \cdots + a_1r^{n-1})$	A gimmick to help simplify
$s - rs =$	$a_1 + a_1r + a_1r^2 + a_1r^3 + \cdots + a_1r^{n-1} +$ $+(-a_1r - a_1r^2 - a_1r^3 - \cdots - a_1r^n)$	Distribute the $-r$
$s - rs =$	$a_1 - a_1r^n$	Almost all terms cancel
$s(1-r) =$	$a_1(1-r^n)$	Factor out GCFs
$s =$	$\dfrac{a_1(1-r^n)}{1-r}$	Divide by $(1-r)$

You do not need to memorize the above proof, or be able to prove the identity on your own, but you do need to memorize that we can use $s = \frac{a_1(1-r^n)}{1-r}$ to find sums of series rather than adding up terms!

Example: Find the sum of $3 + 9 + 27 + 81 + \cdots + 59049$

We either recognize that 59049 is 3^{10} or we more likely have to use our geometric sequence knowledge from the previous lesson to determine this fact.

$$s = \frac{a_1(1-r^n)}{1-r}, a_1 = 3, r = 3, n = 10$$

$$s = \frac{3(1-3^{10})}{1-3} = \frac{3(1-59049)}{-2} = \frac{3(-59048)}{-2} = 88572$$

Expand the series and find the sum:

1) $\sum_{n=1}^{7} 3n - 4$

2) $\sum_{n=0}^{4} -5n$

3) $\sum_{n=5}^{11} (n-1)^2$

4) $\sum_{q=0}^{4} 4(3)^q$

5) $\sum_{p=1}^{5} \frac{3}{10^p}$

6) $\sum_{k=1}^{7} k^3$

7) $\sum_{i=0}^{7} 2(5)^i$

8) $\sum_{j=1}^{8} \frac{7}{2^j}$

9) $\sum_{m=1}^{6} \sqrt{m}$

Write the series using sigma (Σ) notation:

10) $1 + 4 + 9 + 16 + \cdots + 100$

11) $10 + 50 + 250 + 1250$

12) $-1 + 2 + -4 + 8$

13) $1 + -4 + 16 + \ldots + 4096$

14) $20 + 10 + 5 + \ldots + \frac{5}{8}$

15) $-2 + -6 + -18 + \ldots + -162$

16) $-1 + \frac{2}{3} + -\frac{4}{9} + \ldots + \frac{32}{243}$

17) $.16 + .8 + \ldots + 1000$

18) $-128 + 64 + -32$

19) $\frac{2}{5} + \frac{1}{10} + \frac{1}{40} + \ldots \frac{1}{2560}$

20) $.6 + -3 + 15 + -75$

21) $\frac{3}{5} + \frac{9}{5} + \ \ldots \frac{729}{5}$

22) $x^5 + x^4 + x^3 + x^2 + x + 1$

23) $\frac{3}{2} + \frac{3}{4} + \frac{3}{8} + \frac{3}{16} + \frac{3}{32}$

Find the sum of the geometric series by using the geometric sum formula:

24) $2 + 4 + 8 + 16 + 32 + 64 + 128$

25) $\frac{1}{9} + \frac{1}{3} + 1 + 3 + \cdots + 243$

26) $\frac{1}{2} - \frac{1}{3} + \frac{2}{9} - \frac{4}{27} + \frac{8}{81}$

27) $2 + 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \cdots + \frac{1}{1024}$

28) $3 - 1 + \frac{1}{3} - \frac{1}{9} + \cdots + \frac{1}{177147}$

29) $\sum_{n=1}^{7} 3(2)^{n-1}$

30) $\sum_{n=1}^{11} 4(\frac{1}{3})^{n-1}$

31) $\sum_{n=1}^{6} -10(\frac{1}{5})^{n-1}$

32) $\sum_{n=1}^{8} 3(-\frac{1}{4})^{n-1}$

33) You deposit $100 into an account that has 7% monthly compounded interest. What is the account balance after 2 years?

34) Your fridge is running low on milk. The last milk carton has only 1 cup of milk remaining. Your family has a rule that whoever drinks the last of the milk has to go buy more milk, and you don't want that to be you. So each morning, you just drink half of the remaining milk! After a week, your parent notices the game you are playing and then makes you go buy more milk with your own money for being such a pain and breaking the spirit of the house rule. Write the sequence of how much milk you drank each day as sequence m. Write the series s that sums the values of sequence m using sigma notation. Use the series formula to determine how much milk did you drink during the week where you attempted to avoid having to buy more milk?